

# TRANSFORMATIONS OF MESSAGES PROPAGATED IN A SOCIAL NETWORK

*Completed Research Paper*

**Arie Jacobi**

Ono Academic College  
104 Tzahal St.,  
Kiryat Ono, 55000, Israel  
jacobi.arie@gmail.com

**Ofir Ben-Assuli**

Ono Academic College  
104 Tzahal St.,  
Kiryat Ono, 55000, Israel  
ofir.benassuli@gmail.com

## **Abstract**

*As a result of the Web 2.0 information revolution, vast numbers of organizations and individuals communicate by sending messages over social networks. These messages, however, are subject to change as they propagate through the network. This paper calculates the distortion of a message as it propagates in a social network, and compares a global measurement of the distortion in random, scale free and small world networks. We test a mathematical model that we created using a simulation of different types of networks and show that scale-free networks are the least sensitive to distortion of information compared to random or small world networks.*

**Keywords:** social networks, distortion of information, organizational communication, random networks, small-world networks, scale-free networks.

## Introduction

As a result of the Web 2.0 information revolution, vast numbers of organizations and individuals communicate by sending messages over social networks. A message can be sent verbally or using a real-time messaging mechanism. These messages, however, are subject to change as they propagate through the network. This paper calculates the distortion of a verbal message as it propagates in a social network, and compares a global measurement of the distortion in random, scale free and small world networks. We test a mathematical model that we created using a simulation study.

## Background and Literature Survey

This literature survey is divided into two parts. The first briefly presents relevant background from network theory. The second part reviews key studies on distortion and changes in data transmitted through social networks.

### *Propagation and Data Distortion via Social Networks*

Propagation in networks has been studied frequently in the social network community since Rapaport's pioneering study of influence of network characteristics such as transitivity of node linking on disease propagation (Rapaport, 1953a; Rapaport, 1953b).

Propagation or diffusion refers to the transport on a network from node to node of some quantity, such as information, opinion or epidemics. The spread of socially-transmitted diseases is a canonical example (see Newman (2002) for a modeling approach from the theoretical physics perspective and Eames and Keeling (2002) for an up-to-date approach in bio-mathematics, together with references on their study of the propagation of AIDS).

There are many ways to assess a social network as an instrument for information exchange between individuals, groups or organizations. Haythornthwaite (1996) presented a "social network analysis", an approach and set of techniques to study the exchange of resources (information) among actors. Regular patterns of information exchange reveal themselves as social networks, with actors as nodes in the network and information exchange relationships as connectors between nodes. Information exchange relationships structure the flow of information among actors. Social network analysis assesses information opportunities for individuals or groups of individuals in terms of exposure to and control of information. By identifying existing information exchange routes, information providers can act on information opportunities and make changes to information routes to improve the delivery of information services.

There are few works on the perceived quality of information sent by subjects through social networks. O'Reilly (1978) conducted several laboratory and field studies investigating antecedents to and consequences of the intentional distortion of information by senders in organizational communication networks. Laboratory studies were used to examine the impact of two interpersonal variables (trust in the receiver and perceived influence of the receiver over the sender) and directionality of information flow (upward, lateral, and downward) on senders' propensities to block or suppress information. Field studies were used to establish the external validity of the laboratory investigations and to relate information distortion by senders to job satisfaction and performance.

The results of these studies demonstrate that (1) a bias exists towards screening certain types of information from upward transmission; (2) low trust in the receiver of a message results in significantly more suppression of information by senders, especially information which reflects unfavorably on the senders; and (3) a measure of information distortion is significantly and inversely associated with job satisfaction and individual and group performance. These findings differ in several respects from previous studies on organizational communication. Their implications for decision-making are discussed and O'Reilly suggested a model relating antecedents and consequences to information distortion.

More recently, Ma et al. (2010) argued that when a piece of information spreads on a complex network, error or distortion can occur. Information explosion can occur where the number of distinct pieces of information on the network increases continuously with time, leading to high error probability. These authors constructed a physical model to address this phenomenon. They describe the transition to information explosion as the error probability increases through a critical value, and put forward a control strategy to maximize the robustness of the network

against information explosion, which they then validate by both numerical computation and a mean-field based analysis.

The current literature is missing a research about the robustness of different types of social networks against distortion of information.

### Overview of our Method

In this paper we present a study of the robustness of a social network against distortion of information in the form of a verbal message. A message has the tendency to change when one person transfer it verbally to another person. Our assumption is that this change in the transferred message, or its distortion as we call it here, is usually reflected in different parts of the message, but some of the information remains unaltered in the original message. We compare a global measurement of the distortion of the propagated messages in random, scale free and small world networks using a simulation.

The simulation involved a model of the network and a model of the distortion process propagating on the network. The propagation model was tailored so as to reflect the realities of the dissemination of information in a social network.

### Network Background

To calculate the distortion of information transmitted in a social network, we first define some basic terms of graph theory. A network is represented by its mathematical model, the graph. A graph  $G = (V, E)$  formally consists of a set of vertices  $V$  and a set of edges  $E$  between them. An edge  $e_{ij}$  connects vertex  $i$  with vertex  $j$ .

The neighborhood  $S_i$  for a vertex  $v_i$  is defined as its immediately connected neighbors as follows:

$$(1) \quad S_i = \{v_j : e_{ij} \in E \wedge e_{ji} \in E\}.$$

We define  $k_i = |S_i|$ , as the number of vertices in the neighborhood  $S_i$ , of a vertex  $v_i$ .

The local clustering coefficient  $C_i$  for a vertex  $v_i$  is given by the proportion of links between the vertices within its neighborhood  $S_i$  divided by the maximum number of links that could possibly exist in  $S_i$ . For an undirected graph which has the property that  $e_{ij}$  and  $e_{ji}$  are considered identical, if a vertex  $v_i$  has  $k_i$  neighbors, then it is interconnected by up to  $k_i(k_i - 1)/2$  nodes, which is the maximum number of edges within  $S_i$ . Therefore, the local clustering coefficient for undirected graphs is defined as

$$(2) \quad C_i = 2| \{e_{ij} \} | / k_i(k_i - 1) : v_i, v_j \in S_i, e_{ij} \in E$$

On the network level, a network  $N$  contains  $n$  nodes interconnected by up to a total of  $m = n(n - 1)/2$  edges. Each node represents a person and each edge that connects two nodes represents a connection between two people. In the networks we use in this research, there is at most only one edge between any two nodes and no node connects to itself.

The connectivity probability  $p$  of a network is the average probability that any node is connected to any other node. It is simply the number of edges divided by the maximum possible number of edges, or

$$(3) \quad p = \frac{2m}{n(n - 1)}.$$

A fully connected network has a probability of  $p = 1$ .

In the past few years, the discovery of small-world and scale-free properties of many natural and artificial complex networks has stimulated a great deal of interest in studying the underlying organizing principles of various complex networks, which has led to dramatic advances in this emerging and active field of research (Wang and Chen, 2003). Wang and Chen (2003) review some basic concepts, important progress, and significant results in current studies of

various complex networks, with an emphasis on the relationship between the topology and the dynamics of such complex networks. Some fundamental properties and typical complex network models are described; and, as an example, epidemic dynamics are analyzed and discussed in some detail.

## Research Objectives

We calculate the distortion of a verbal message as it propagates in a social network. Additionally, we compare the results of a global measurement of the distortion in random, scale free and small world networks, after propagation of the message in the network, to examine the robustness of each type of network against distortion of information. Following the conclusions of this research, we present a practical recommendation for how to prevent information distortion which is diffused in social networks.

## Methodology

### Proposed Model

Given:

- A network  $N$  with  $n$  nodes
- A message  $m$  can represent a sequence of letters, words, or parts of sentences. Without loss of generality in this model we chose to define  $m$  as is a sequence of letters  $\sigma_i \in \Sigma, i = 1, \dots, k$ , where  $\Sigma \in \{0,1\}$  is an alphabet. We define the length  $k = |m|$  of  $m$  to be number of letters in  $m$ . We can refer to  $m$  as a Boolean vector  $\mathbf{v} = [\mathbf{v}_0, \dots, \mathbf{v}_k]^T$  of degree  $k$ , where  $\mathbf{v}_i \in \{0,1\}$ .

The message  $m$  represents a verbal message that a person sends to other individuals over a social network. Therefore, each letter  $\sigma_i$  represents a letter in a natural language message.

Initially, the message  $m$  is transmitted verbally by  $l$  different people in network  $N$ . These people forward  $m$  to some or to all of the people they know (adjacent nodes in the graph that represent  $N$ ). At every transmission of  $m$  from a given person  $P_i$  to person  $P_j$  in network  $N$ ,  $m$  may mutate (be distorted) into  $\hat{m}$ , such that some of the letters in  $m$  (chosen randomly) will change value. The number of letters that change can differ from person to person in the network. The mutated message  $\hat{m}$  can continue mutating each time a person  $P$  receives a new message  $\hat{m}$  from another person in the network.

In order to create a mutated message  $\hat{m}$ , we need to consider all the  $r$  messages received by person  $P$ .

$$(4) \quad (m_1^P, \dots, m_r^P),$$

For every mutated letter  $\sigma_i \in \hat{m}, i = 1, \dots, k$ ,  $\sigma_i$  is chosen to be the letter that has the maximum number of occurrences among all letters (the mode) at location  $i$  in  $(m_1^P, \dots, m_r^P)$ . In the case where we have an equal number of different letters the original letter in  $m_1^P$  is chosen.

Let

$$c^1 = \text{number of "1"s in all letters } \sigma_i \in (m_1^P, \dots, m_r^P),$$

And let

$$c^0 = \text{number of "0"s in all letters } \sigma_i \in (m_1^P, \dots, m_r^P).$$

We calculate

$$(5) \quad \sigma_i = \begin{cases} 1, & \text{if } c^1 > c^0 \\ 0, & \text{if } c^1 < c^0 \\ \sigma_i \in m_1^P & \text{if } c^1 = c^0 \end{cases}.$$

For example, if we have the following four consecutive messages, the final mutated message will be:

Message $m_1^P$	0 1 0 1 0 1 1
Message $m_2^P$	0 0 0 1 1 0 1
Message $m_3^P$	0 1 1 0 0 1 1
Message $m_4^P$	0 1 0 0 1 1 1
Final mutated message $\hat{m}^P$	0 1 0 1 0 1 1
<b>Figure 1. Example for a Mutated Message</b>	

since a person  $P$  can forward the message  $\hat{m}^P$  to other people and subsequently can receive additional messages that can affect his original message, the calculation of  $m_i^P$  at each such stage is done according to (5). This is shown in the following example:

Message $m_1^P$	0 1 0 1 0 1 1
Message $m_2^P$	0 0 1 1 1 0 1
$P$ sends message	0 1 0 1 0 1 1
Message $m_3^P$	0 1 1 0 0 0 1
$P$ sends message	0 1 1 1 0 0 1
Message $m_4^P$	0 1 0 0 1 0 1
Final mutated message $\hat{m}^P$	0 1 0 1 0 0 1
<b>Figure 2. Example for a Mutated Message</b>	

### Operationalization of the Research Variables

#### Dependent Variables

Two types of errors are measured: **relative error** and **absolute error**. The relative error  $E^i$ , for a person  $P^i$ , represents the number of mutations from the original message  $m_1^P$  that person  $P^i$  initially received. It is calculated as follows:

Let  $\mathbf{u} = \hat{m}^P - m_1^P$  be the difference vector of the messages of a person  $P^i$ . We calculate

$$(6) \quad E^i = \text{Number of "1"s in } \mathbf{u}^1.$$

For example, take the messages in Figure 2. We calculate the difference vector:

$$\mathbf{u} = \hat{m}^P - m_1^P = [0101011]^T - [0101001]^T = [0000010]^T.$$

Therefore,  $E^i = 1$  which is the number of "1"s in  $\mathbf{u}$ .

The absolute error  $EA^i$ , for a person  $P^i$ , represents the number of mutations from the original message  $m$  that was first propagated. It is calculated as follows:

Let  $\mathbf{u} = m - m_1^P$  be the difference vector for person  $P^i$ . We calculate

$$(7) \quad EA^i = \text{Number of "1"s in } \mathbf{u}.$$

Example: Assuming that the original message  $m = [1101111]^T$  and by taking the final message in Figure 2, the difference vector:

$$\mathbf{u} = m - m_1^P = [1101111]^T - [0101001]^T = [1000110]^T.$$

Therefore,  $EA^i = 3$  which is the number of "1"s in  $\mathbf{u}$ .

After the propagation of  $m$  in network  $N$  that contains  $n$  people, we can then calculate the average global relative distortion value  $N_R^D$  for  $N$  as

$$(8) \quad N_R^D = \sum_{j=1}^n E^j / n,$$

and calculate the average global absolute distortion value  $N_A^D$  for  $N$  as

$$(9) \quad N_A^D = \sum_{j=1}^n EA^j / n.$$

## Independent Variables

**Degree** - The degree of a vertex in a network is the number of edges attached to it. Degree is often interpreted in terms of the immediate risk that a node will contract whatever is flowing through the network (such as a virus, or some information).

**Type of network** – Three types of networks are analyzed in this paper:

- Random networks (Erdős and Rényi, 1959; Gilbert, 1959), in which the nodes are randomly interconnected by a number of edges with probability  $P_{ER}$ .
- Small world networks (Watts and Strogatz, 1998), in which the nodes are initially evenly interconnected, with each node connected to its nearly adjacent nodes. The edges are then randomly rearranged with probability  $P_{WS}$ .
- Scale-free networks (Barabási and Albert, 1999; Albert and Barabási, 2002), in which the network starts with  $m_0$  unconnected vertices, and at each time step  $t$ , another node is added with  $m$  edges ( $m \leq m_0$ ). The

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<sup>1</sup> The vector  $\mathbf{u}$  can efficiently be created by using the logical operator AND instead of subtraction.

probability  $\Pi_i$  of existing node  $i$  being connected to the new node is proportional to the connectivity of that node.

$$(10) \quad \Pi_i = k_i / \sum_j k_j$$

Preference is thus given to "earlier" nodes, thereby forming hubs.

**Hub** – Scale-free networks characterized by a power law distribution of the number of links connecting to a node, and therefore include nodes which are often called "hubs", which have many more connections than others. In graph theory terms these nodes (vertices) have a degree that exceeds the average degree by an order of magnitude (e.g., Valente, 1996; Barabási and Crandall, 2003).

**Original Message** – The original message is the message that was first to propagate in the network.

**First Propagator** – Is a person (node) in the network that holds the original message and the first to propagate it in the network. There might be a number of First Propagators of the original message.

**Message-In-Memory** – Message-In-Memory is the message that a person in the network (node) holds. This message was formed in two possible ways:

- It was initially received from one of this person's connections in the network, and therefore, will be identical to this initial message, which is called the "Person-Original-Message".
- It already exists, but it is affected and changed by other messages that the person receives from his/her connections in the network. Every letter of the newly affected Message-In-Memory is calculated as the mode (most frequently occurring) of the letters at the same location in all the previous messages that this person received.

**Transferred message** – Before transfer of Message-In-Memory, the message will be distorted by the person and then it is transferred to some or all of his/her connections in the network.

**Mutation** – Every time a message propagates in a network, and is transferred from one person (the sender) to another (the receiver), it is distorted by the sender of the message. The receiver gets a mutated message and each such a message is called a mutation.

## Model Simulations

We test the model on scale free, small world and random networks to determine which type of network is the least sensitive to distortion of information. We also calculate the two types of errors (7,8). Finally, we compare the statistical results using mathematical and statistical tools.

The algorithm we use to traverse the undirected graph that represents the network under simulation is Breadth First Search (BFS) algorithm for graph search and traversal.

### Breadth First Search (BFS) Algorithm

Search in a graph is defined as finding a path from a start node to a destination node. The cost of a search is the number of edges traversed in locating the destination node (i.e., the number of "messages" sent in the network during the search process).

Breadth First Search (BFS) is a search algorithm, defined to find a path from a start node to a destination node in a graph. We use BFS to traverse a given network which is represented using a data structure of graph. BFS begins at the source node (First Propagator) by checking each of its neighbors. Each of these neighbors checks their neighbors and this continues until all the nodes were visited.

We slightly changed the basic BFS algorithm to include random selection of neighbors, and to allow more than one visit at each node. At each node  $v$  that BFS visits, BFS is checking if the number of visits did not exceed a given

limit of allowed visits per node. BFS is also checking  $n$  neighboring nodes which are chosen randomly, and for each of them, the algorithm takes the "message-in-memory" of node  $v$ , mutates it, and transfers it to each one of the  $n$  randomly chosen neighbors. The mutated message will affect each of  $v$ 's neighbors, and is done according to equation (4, 5) above.

The BFS algorithm is executed for each First Propagator separately and sequentially. Each First Propagator leaves its impressions on every node in the network accumulatively. In other words, the effect the first of the First Propagators leaves on the messages at each node in the network remains when the second First Propagator start to propagate and the algorithms take into account the previous information at the nodes (the messages that the node received during the propagation executed by the first of the First Propagators).

## **Data Description**

In this research we used network analysis package NWB developed by Barabási's team at Indiana University (NWB Team, 2006), and self written software to produce the different types of networks. The scale-free networks we tested in our simulations came with a slope of -2 to -3, which characterizes human social networks. In addition, we determined which node is a hub using the three sigma criteria. The results presented in this research used only one First Propagator, each node in the network could be visited twice, and each simulation on a given network ran one time. At each run, the neighbors of each visited node were chosen randomly. In addition, the letters that went through mutation during the distortion process of a "transferred message", were also chosen randomly.

## **Research Hypotheses**

H1: Scale free networks are less sensitive to data distortion than Random networks.

The resulting two sub-hypotheses:

H1.1: The relative error will be lower in Scale free networks than in Random networks.

H1.2: The absolute error will be lower in Scale free networks than in Random networks.

H2: Scale free networks are less sensitive to data distortion than Small world networks.

The resulting two sub-hypotheses:

H2.1: The relative error will be lower in Scale free networks than in Small world networks.

H2.2: The absolute error will be lower in Scale free networks than in Small world networks.

Scale-free networks are less sensitive to data distortion than small worlds, or random networks. The scale-free networks are characterized by a power law distribution of the number of links connecting to a node, and therefore, the existence of hubs. Our hypotheses are based on the scale free network structure, such that hubs and high degree nodes receive a message at an early stage of the propagation, and they deliver it to many people in the network (Rapaport, 1953a; Rapaport, 1953b; Barabási and Albert, 1999; Albert and Barabási, 2002). Therefore, the message path through a scale free network is shorter than the other types of networks, and as a result fewer message distortions occur in the propagation process.

## **Findings**

### ***Data Breakdown***

We present the descriptive statistics for each of the three types of networks. For each network we ran the simulation twice with different numbers of nodes.



### Scale Free Network

<b>Table 1. Descriptive Statistics for Scale Free Network with 30,000 nodes</b>					
	Degree	Mutations	Hub	Rel_Err	Abs_Err
Mean	4.00	.47	.00	.22	3.33
Median	3.00	.00	.00	.00	4.00
Mode	2	0	0	0	0
Std. Dev	6.579	.675	.006	.783	2.622
Range	1-402	0-2	0-1	0-8	0-10

### Random Network

<b>Table 2. Descriptive Statistics for Random Network with 30,000 nodes</b>				
	Degree	Mutations	Rel_Err	Abs_Err
Mean	20.09	.62	.28	4.17
Median	20.00	.00	.00	4.00
Mode	19	0	0	5
Std. Dev	4.490	.687	.900	2.176
Range	5-43	0-2	0-7	0-10

### Small World Network

<b>Table 3. Descriptive Statistics for Small World Network with 30,000 nodes</b>				
	Degree	Mutations	Rel_Err	Abs_Err
Mean	40.0	.66	.25	4.41
Median	40.0	1.00	.00	5.0
Mode	40	0	0	5
Std. Dev	1.962	.699	.812	2.16
Range	32-50	0-2	0-8	0-10

### Statistical Analysis

Statistical analyses were performed using SPSS version 18 (SPSS Inc., Chicago, IL, USA) software. To test for differences in continuous variables between the two groups a t-test for independent samples or a Mann-Whitney U test were performed.

To test for differences in continuous variables between more than two networks a one-way analysis of variance (abbreviated one-way ANOVA) was performed. This technique is used to compare means of two or more samples (using the F distribution) for numerical (parametric) data (Anderson et al., 2009; Tabachnick et al., 2007).

Associations between dichotomous variables were tested with the Pearson Chi-Square test (the standard test to compare proportions) or Fisher's Exact Test.

### Comparing the Difference between the Means of the Errors

We used a 1-way ANOVA test to explore for differences in the means of the relative error and the absolute errors.

Table 4. One- Way ANOVA for the larger network size (N=30,000)						
		Sum of Squares	df	Mean Square	F	Sig.
Rel_Err	Between Groups	43.225	2	21.612	31.802	.000
	Within Groups	54364.969	79997	.680		
	Total	54408.194	79999			
Abs_Err	Between Groups	18961.948	2	9480.974	1719.786	.000
	Within Groups	441013.832	79997	5.513		
	Total	459975.780	79999			

In this sample of nodes, there were also differences between the means of the three networks for both of the errors (relative error:  $F=31.8$ ,  $P<0.001$  and absolute error:  $F=1719.79$ ,  $P<0.001$ )

In the following results, we also tested for the differences in each pair of networks to analyze the connections between the networks and to test our hypotheses.

### Testing Our Hypotheses

In order to test our hypotheses we compared averages between networks using T-tests. We tested both types of errors and present them in the Tables 5 – 6 below.

#### Random Networks vs. Scale Free (H1)

Table 5. Testing hypothesis H1								
	Relative Error (H1.1)				Absolute Error (H1.2)			
N	Random Network (stdev)	Scale Free (stdev)	T-Test-value (df)	p-value	Random Network (stdev)	Scale Free (stdev)	T-Test-value (df)	p-value
30,000	0.28 (0.900)	0.22 (0.783)	7.688 (38688.2)	<0.001	4.17 (2.176)	3.33 (2.622)	39.093 (47675.9)	<0.001

#### Small World vs. Scale Free (H2)

Table 6. Testing hypothesis H2								
	Relative Error (H2.1)				Absolute Error (H2.2)			
N	Small World (stdev)	Scale Free (stdev)	T-Test-value (df)	p-value	Small World (stdev)	Scale Free (stdev)	T-Test-value (df)	p-value
30,000	0.25 (0.812)	0.22 (0.783)	3.722 (59917.4)	<0.001	4.41 (2.161)	3.33 (2.622)	55.203 (57889)	<0.001

### Summary of the Results:

- We ran the simulation several times to increase the robustness of our results and obtained very similar results. The generality of our conclusions and their applicability to other sizes of social networks must be considered with the appropriate degree of caution. In general, we found differences between the means of the three networks regarding both types of errors:
- The relative error is lower in scale free networks than in random networks (H1.1 accepted). Additionally, the absolute error is lower in scale free networks than in random networks (H1.2 accepted).
- The relative error is lower in scale free networks than in small world networks. (H2.1 accepted). Additionally, the absolute error is lower in scale free networks than in small world networks (H2.2 accepted).

### Conclusions and Discussion

Scale-free networks are less sensitive to data distortion because of the existence of hubs in scale-free. The hubs and the high degree nodes receive a message at an early stage of the propagation, and they deliver it to many people in the network. Therefore, the message path through the network is shorter than the other types of networks, and as a result fewer message distortions occur in the propagation process.

An immediate result from the above conclusion, is when one would like to minimize the distortion of a message in a social network, one should try to pass it as soon as possible to the hubs in the network, and if the network does not include hubs, then pass the message to the nodes with the highest degree. This recommendation will not be practical for random networks and a different method should be applied in this case.

### Avenues for Future Research

Future research will work to remedy the limitations of the current research and enhance the validity of the conclusions of this study.

- Increase the sample size, which is currently not large enough. The right size for the scale-free networks for our research purposes starts at 100,000 nodes.
- Test more independent variables that relate to each node in the network and globally to the whole network. Examples of such variables are:

- Degree Centrality - For a network  $N$  with  $n$  vertices, the degree centrality  $C_D(v)$  for vertex  $v$  is defined as

$$C_D(v) = \deg(v)/n - 1$$

Previous works already discussed on this variable (Coleman et al., 1966; Bavelas, 1948; Freeman, 1979; Rogers and Kincaid, 1981).

- Betweenness centrality (Newman, 2005) - Betweenness is a centrality measure of a vertex within a network. Vertices that occur on many of the shortest paths between other vertices have higher betweenness than those that do not. Betweenness is a crude measure of the control over the flow of information. If we imagine information flowing between individuals in the network and always taking the shortest possible path, then betweenness centrality measures the fraction of that information that will flow through a vertex  $v$  on its way to wherever it is going.
- Extent of trust in information passed to individuals from different sources in the network. It is worth investigating the integration and the mutual relationships of both trust and distortion of information.

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